Solar Flare 1/f Fluctuations from Amplitude Modulated Five Minute Oscillation

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We first study the solar flare time sequence based on the GOES16 data. We find that the power spectrum density of the low-energy ($E \leq E_{mean}$) flare shows 1/f fluctuations, but the high-energy ($E > E_{mean}$) flare shows a flat spectrum. Further, we found that the flare timing time-sequence shows 1/f fluctuations clearer. These facts indicate that the solar flare 1/f fluctuations are associated with low-energy phenomena. We investigate the origin of this 1/f fluctuations based on our recent proposal: 1/f fluctuations arise from amplitude modulation and demodulation. We speculate that this amplitude modulation is encoded by the resonance with the Solar Five-minute Oscillation (SFO) and demodulated by magnetic reconnection.We partially demonstrate this scenario by analyzing the SFO eigenmodes resolving the frequency degeneracy in the azimuthal order number m by solar rotation and resonance. Since 1/f fluctuation is robust, we speculate that the solar flare 1/f fluctuations may be inherited by the various phenomena around the Sun, such as the sunspot numbers and the cosmic rays. Finally, we compare the solar flares and the earthquakes, both showing 1/f fluctuations. Interestingly, the same analysis for solar flares is possible for earthquakes if we read SFO as Earth's Free Oscillation, and magnetic reconnection as fault rupture. Furthermore, we point out the possibility that the same analysis also applies to the activity of the black hole/disk system if we read SFO as the Quasi-Periodic Oscillation of a black hole.

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I. INTRODUCTION

A Solar flare is a sudden energy eruption in the solar atomosphere[1]. It is triggered by magnetic reconnection, and the enormous magnetic energy $10^{17} - 10^{26}$ J is converted into plasma particle acceleration, heating, and light emission. Solar flares are quite complex phenomena, and the statistical approach is effective, as in the case of earthquakes, being a sudden energy eruptions in the Earth's crust.

It is well known that solar flares and earthquakes are similar to each other, and they show similar statistical properties. In particular, scaling relations, such as the Gutenberg-Richter law[2] and the Omori law[3], are universal laws for both solar flares and earthquakes [4, 5].

Here in this paper, we concentrate on the solar flare and would like to add one more universal scaling law in the ultra-low frequency region of the power spectrum density (PSD) for the long time-sequnce of the solar flares. It turns out that the solar flare time sequence shows the power law almost inversely proportional to the frequency in PSD. This is often called 1/f fluctuation or pink noise and appears in most fields in nature and human activities[6, 7]. However, the origin of this fluctuation was not clarified despite tremendous studies in the past century.

We recently proposed that the general origin of pink noise is amplitude modulation (AM), or the beat of many waves with accumulating frequencies[8]. In particular, this frequency accumulation is possible in resonance where many eigenfrequencies are systematically concentrated in a narrow domain.

Applying this method in [9], we studied pink noise in the seismic activities. The seismic-energy time sequence shows an apparent pink noise in its PSD in more than three digits if giant earthquakes are excluded. Therefore, seismic pink noise is considered to be associates with low-energy phenomena. In this case, perpetually exciting Earth Free Oscillation (EFO) in the lithosphere is resonating to yield AM or wave beats. Relatively low energy EFO will sufficiently trigger the fault ruption and cause earthquakes.

In this paper, applying this proposal to solar flares, we would like to verify our proposal and try to figure out the statistical properties of the complex systems in general. When we naively use all the energy time series data, we obtain almost flat PSD at low-frequency regions and obtain no clear pink noise. However, if we restrict the solar flare events with their energy below the mean, we obtain clear pink noise. Therefore, we speculate that the pink noise in

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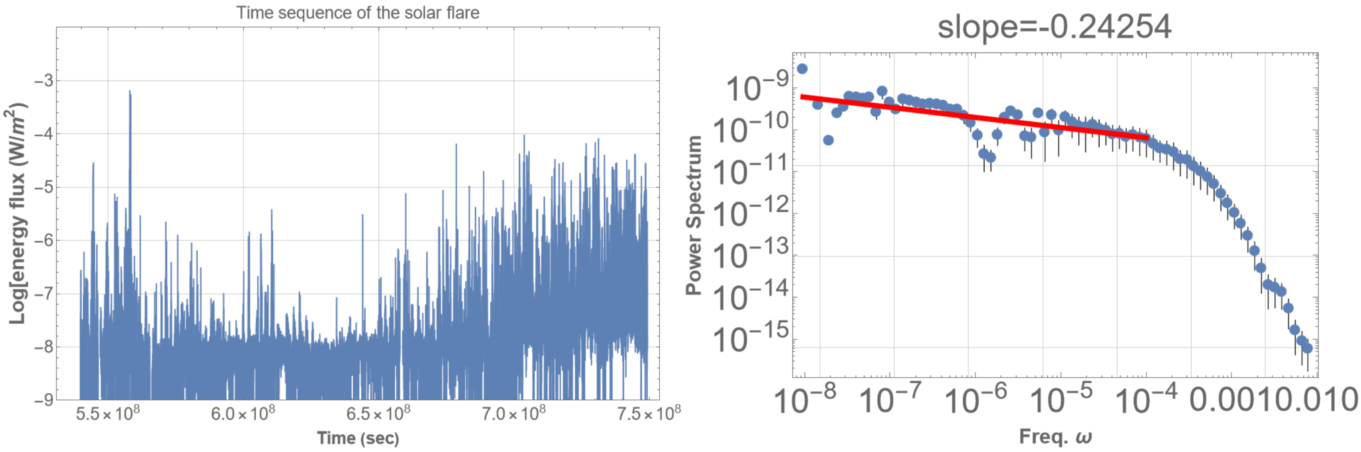

Figure 1. left) Goes16 soft X-ray flux data (February 2017 to September 2023, 6.6 years)[12]. The unit is W/m^2 in the logarismic scale. Although the data shows an apparent trend associated with the eleven-year solar activity, we do not apply any artificial operation for our analysis; extraction of the trend does not greatly affect the result.
right) Power Spectrum Density (PSD) of the energy-flux time-sequence of all the GOES16 solar flare data 6.6 years. The time is measured in seconds, and the frequency unit is Hz. Before the analysis, the data are homogenized over time. This is the same for all PSD analyses below. This PSD shows that the energy time sequence is random, as shown by the almost flat red line that fits the data points in the low-frequency domain.

solar flares is associated with low-energy phenomena. Interestingly, this situation is the same as the seismic activity, as explained above.

The resonation in the solar case would be characterized by the solar five-minute oscillations (SFO), which is perpetually excited by the turbulence in the solar convective region[10, 11]. The eigenfrequencies are precisely measured and calculated by many studies assuming appropriate solar models. Using this observational data, we construct the superposition of waves with these eigen-oscillations, including the fine splitting structure by the solar rotation and the resonance effects. Then, we can obtain pink noise from the square of this data in PSD. Thus, we can partially demonstrate the AM theory for pink noise in solar flare.

Since pink noises often appear in any field of science, we explore the neighboring phenomena to the solar flare. Then we find several phenomena such as solar wind, sun spot numbers, and some traces on Earth. These facts may indicate the robustness of pink noises.

The construction of this paper is as follows. In the next section 2, we explore the GOES data of solar flares and analyze PSD. In section 3, we explain the AM proposal for the origin of pink noise from resonance. In section 4, we superpose the eigenmodes of SFO and obtain pink noise. In section 5, we study another statistical characterization of the solar flare by the Weibell distribution and compare it with the 1/f characterization. In section 6, we emphasize the robustness of pink noise is the origin of the variety of pink noises. We point out that the pink noise property is inherited by solar winds, sunspot events, and some traces on Earth. In section 7, we conclude our work and briefly describe the possible future research.

II. SOLAR FLARE FLUCTUATIONS

Solar flares are eruptive energy-release events in the solar atmosphere. In each event, enormous magnetic energy is transformed into plasma particle acceleration, visible light, X-ray, etc. Our focus is specifically directed towards the soft X-ray flux data acquired by the GOES16 satellite during the timeframe spanning from February 2017 to September 2023, encompassing a duration of 6.6 years.[12].

We first naively use all the time sequence data of the soft X-ray energy flux in the unit W/m^2, Fig. 1left. The corresponding PSD becomes almost flat and random in the low-frequency regions, as in Fig. 1 right. However, this is consistent with the previous research[13], in which the authors of [13] partially extract 1/f fluctuations in the GOES6 data by superposing multiple PSDs. We try another approach to extract the entire 1/f fluctuation by restricting energy flux.

Next, we do the same analysis for the two data sets: the high-energy group, which include all the event with energy larger than the mean, and the low-energy group, which include all the event with energy smaller than the mean. PSDs are shown for these two data sets in Fig. 2. It is apparent that the pink noise does appear in Fig.2 right, the

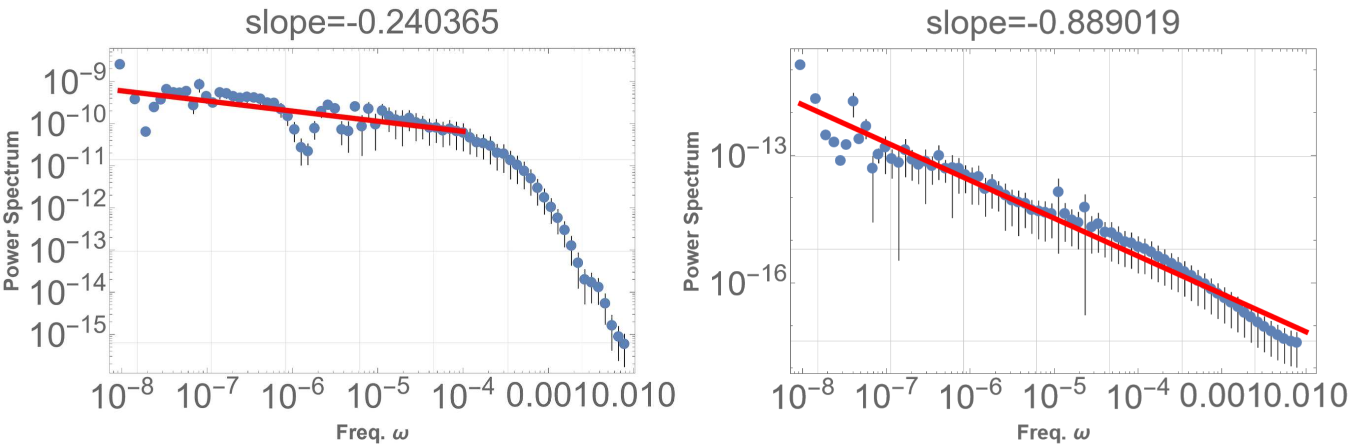

Figure 2. left) PSD for the high-energy group, which include all the event with the energy larger than the mean. The PSD does not show 1/f fluctuations, similar to Fig.1 right.
right) PSD for the low-energy group, which include all the event with the energy smaller than the mean. The PSD shows clear pink noise with an index of -0.89 in more than five digits.

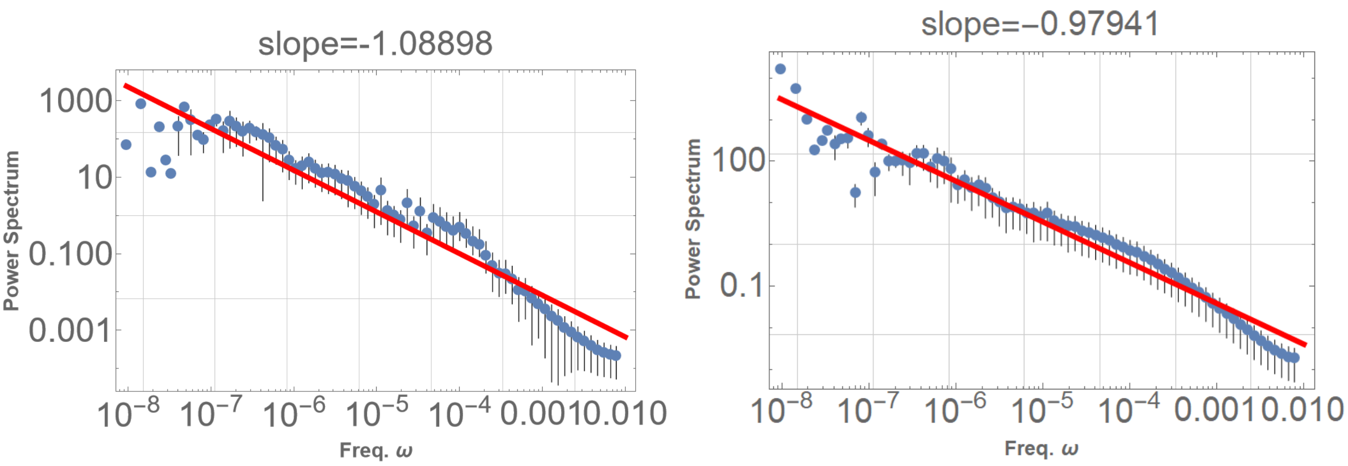

Figure 3. left) Same as Fig.1, but the energy information is fully removed; we reset the energy value to one for each data. The PSD shows the power behavior with an index of -1.1, a typical pink noise, in the range 10^{-3}to2×10^{-8} Hz.
right) Same as left PSD but in the high-energy group. This gives an apparent contrast with the left of Fig.2, in which all the energy information is included.

low-energy data. On the other hand, PSD for the high-energy group does not show 1/f fluctuations as in Fig.2 left, similar to Fig.1 righr; high-energy data destroys the 1/f fluctuations in the solar flare. These facts indicate that the solar flare pink noise is associated with low-energy phenomena.

In order to confirm that the solar flare 1/f fluctuation is independent from energy, we entirely remove the energy information from the data: we set all the energy values in the time sequence to one. Then, the PSD for the entire data turns out to show pink noise with the power of -1.1 as in Fig3 left. Even the PSD for the high-energy group, if energy information is likewise removed, shows pink noise with the power of -0.98 as in Fig3 right.

All together, pink noise with the power index $-0.9 \sim -1.1$ is observed within about five digits of frequencies corresponding to the timescales from about an hour (10^{-3} Hz) to 6.6 years(2×10^{-8} Hz). This pink noise appears when we remove energetic solar flare events or fully remove the energy information.

These facts indicate that the solar flare pink noise does not reflect the energy scaling structure typically caused by the self-organized criticality (SOC) formed by energy cascades from small to large, although SOC may be crucial to explain the popular scaling laws Gutenberg-Richter and Omori laws. Contrary, the above facts indicate that the solar flare pink noise is a low-energy phenomenon, probably triggered by a tiny energy source.

We have also analyzed short-term GOES16 solar flare data for one week, arbitrarily chosen. The results are consistent; showing the presence of pink noise with an index of -1.15 within the frequency range of $2 \times 10^{-3}$to

2×10^{-6} Hz. This extends across the entire week, encompassing frequencies lower than the typical frequency of SFO.

We have further analyzed RHESSI solar flare 16 years data, 2002-2018[14]. The results are fully consistent, showing the presence of pink noise with an index of -1.0 within the frequency range of $5 \times 10^{-6}$to 2×10^{-9} Hz, if we ignore the energy information. This extends from several days up to the whole observation period.

What mechanism then gives rise to this universal pink noise at the low-energy regime of various solar flare data?

III. AMPLITUDE MODULATION FROM RESONANCE

We recently proposed a potential origin for 1/f fluctuation, attributing it to amplitude modulation [8]. Given the generality of this mechanism, our intention is to extend its application to the context of solar flare pink noise within the scope of this paper.

The foundation of this theory rests on the observation that waves with accumulating frequencies yield robust low-frequency signals. In cases where this accumulation is systematic, such as in instances of resonance, synchronization, or infrared divergence, the beats consistently manifest a power law with an index approximately equal to -1. The ubiquity of this phenomenon in nature stems from the prevalence of simple physics governing beat or amplitude modulation.

Consider the example of a beat: waves with frequencies 440 Hz and 441 Hz yield a beat. In musical sounds, this beat is 'audible' as a sinusoidal amplitude oscillation with a frequency of 1 Hz. However, Fourier Transform does not yield a 1 Hz signal, only the original two frequencies. To extract this encoded 1 Hz signal, a simple method is to square the data and then Fourier Transform. This allows for the extraction of the encoded low-frequency signal at 2 Hz, though twice the original. Decoding is not limited to squaring but can also involve the absolute value, rectification, 4th order power, thresholding, or other methods, resulting in a variety of pink noise.

Another example is AM radio, where Amplitude Modulation is utilized. Using high-frequency radio waves of 526.5 kHz to 1606.5 kHz, a low-frequency audible signal is encoded. However, the encoded sound cannot be heard directly, as rapidly oscillating positive and negative parts in the wave cancel each other out, leaving no audible signal. Demodulation is achieved by rectifying the radio wave signal, typically through the use of germanium diodes or vacuum tubes. This process is indispensable for extracting the encoded low-frequency signal, such as pink noise [8].

In the context of pink noise in solar flares, we hypothesize that the resonant mode crucial for the manifestation of 1/f fluctuations is the Solar Five-minute Oscillation (SFO), a phenomenon consistently activated within the solar atmosphere through turbulent convection[10, 11]. Specifically, pressure modes of SFO exhibit accumulating eigenfrequencies, particularly converging towards lower angular indices l. We aim to examine whether this frequency accumulation effectively produces a 1/f power spectrum density.

If the SFO induces amplitude modulation, demodulation becomes imperative for the observation of 1/f fluctuation to occur [8]. This necessity arises due to the cancellation of positive and negative components within the relatively high-frequency wave, encompassing 1/f modulation. In the context of solar flares, the demodulation process is envisioned to be facilitated by the threshold established through magnetic reconnection. The tiny energy required for magnetic reconnection may have a 1/f fluctuation characteristic, aligning with the tiny energy associated with SFO that can trigger solar flares.

Subsequently, we apply this theory of amplitude modulation to the analysis of 1/f fluctuations in solar flares.

IV. RESONATING SOLAR FIVE-MINUTE OSCILLATION

We delve into the potentiality of the Solar Five-minute Oscillation (SFO) as a catalyst for 1/f fluctuation in solar flare activity. Specifically, our focus centers on elucidating how SFO eigenmodes contribute to the accumulation of frequencies, thereby generating low-frequency signals through amplitude modulation mechanisms.

The small displacement, denoted as $u(t, r, \theta, \phi)$, of the solar atmosphere from its equilibrium position follows the Poisson equation

$$\rho \ddot{u} = \kappa \triangle u - \rho \nabla \phi_g, \tag{1}$$

where $p, \rho, \kappa, G, \phi_g$ represent the pressure, mass density, bulk modulus, gravitational constant, and gravitational potential, respectively.

The stationary solution $u(t, r, \theta, \phi) = v(r, \theta, \phi)e^{-i\omega t}$ leads to the eigenvalue equation. Utilizing the variable separation method in the spherical coordinate system, we obtain a solution of the form

$$u(t, r, \theta, \phi) = R_{n,l,m}(r) Y_l^m(\theta, \phi) e^{-i\omega_{n,l,m} t}, \tag{2}$$

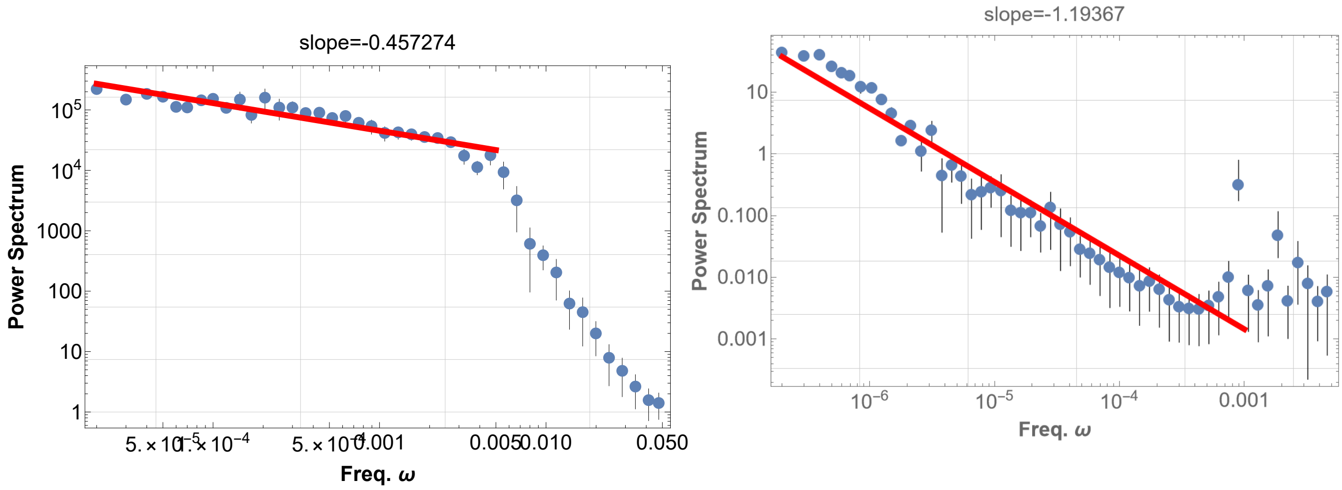

Figure 4. left: On the left side, the graph depicts the Power Spectrum Density (PSD) of the absolute value of the time sequence given by Eq.3, denoted as $|\Phi(t)|$. Here, $\Phi(t)$ represents the superposition of sinusoidal waves with the $N = 2247$ eigenfrequencies of the Solar Five-minute Oscillation (SFO), each with a random amplitude. Each mode is identified by the parameters n, l, and the azimuthal order number m, which is degenerate. Despite exhibiting a power law, this presentation barely demonstrates the characteristics of pink noise.
right: On the right side, analogous to the left graph, this graph includes resonant modes and fine eigenmodes after resolving the degeneracy in m. In constructing the data $\Phi(t)$, we superimpose sinusoidal waves with 100 frequencies from the lowest and introduce $N = 100$ Lorentzian-distributed modes. The latter are randomly generated following Eq.4. The graph displays the PSD of the thresholded value of the time sequence Eq.6,$|\Phi(t)|$. The threshold is strategically set to select data points $|\Phi(t)|$ that surpass twice the mean, although insensitive to the demodulation method. This presentation reveals nearly 1/f fluctuation with an index of -1.2 spanning over four digits. Notably, variations in thresholds and sample sizes in the PSD analysis consistently yield similar 1/f fluctuations.

where $Y_l^m(\theta, \phi)$ represents spherical harmonics, and the modes are characterized by $n = 0, 1, 2, ...,$ $l = 0, 1, 2...,$ and $-l \leqq m \leqq l$. The modes are further categorized as pressure and gravitational modes. All parameters are uncertain depending on the detail of the solar interior, making the solution of the eigenvalue equation a complex task. Numerous numerical calculations and observational studies have been conducted on the aforementioned eigenmode equations.

We utilize observational data pertaining to eigenmodes of solar oscillations obtained through helioseismology [15]. This dataset provides valuable information on numerous observed frequencies, disregarding the degeneracy in the azimuthal order number parameter m $(-l \leq m \leq l)$. A distinctive characteristic of these modes is the accumulation of frequencies towards smaller values of l for each n parameter, typically around 3×10^{-3}Hz. This property is pivotal for the emergence of pink noise through the amplitude modulation mechanism [8].

To simulate the phenomenon, we randomly superimpose all sinusoidal waves with frequencies ranging from the lowest at 848.241μHz up to 4669.16μHz. The wave mode superposition is expressed as

$$\Phi(t) = \sum_{k=1}^{N} \xi_k sin(2\pi\Omega_k t), \tag{3}$$

where ξ_k is a random variable within the range $[0, 1]$, and $N = 2247$ represents the total number of eigenfrequencies in the data. Subsequently, we conduct Fourier analysis (FFT) on the power spectrum density (PSD) for the time series of the absolute value $|\Phi(t)|$. Notably, calculating PSD for the bare $\Phi(t)$ yields no signal in the low-frequency domain. As the 1/f fluctuation signal is modulated in our model, a demodulation process is imperative; taking the absolute value is a typical demodulation method, essential for extracting 1/f fluctuations in the PSD analysis. The specifics of the demodulation process will be explored further in subsequent discussions.

The PSD analysis results in a power-law with an index of approximately-0.5 within the low-frequency range of $2 \times 10^{-5} - 5 \times 10^{-3}$ Hz, as illustrated in Fig. 3 left. However, it is noteworthy that the observed solar flare pink noise occurs in the range of $2 \times 10^{-8} - 10^{-3}$ Hz, considerably lower than our analysis. This disparity suggests the need for a more nuanced consideration of realistic fine structures of the eigenstates and additional resonances, a facet we will delve into in the subsequent analysis.

Our analysis of resonance is currently incomplete, with several aspects requiring further exploration. Firstly, a) each eigenmode is associated with a resonance curve, and numerous frequency-accumulating modes are linked to each mode. Secondly, b) the degeneracy in the azimuthal order number m should give rise to a fine structure around each principal frequency characterized by n and l. This degeneracy in m is resolved by the solar non-spherical symmetry or the solar rotation. In this paper, we examine representative modes for both cases (a and b) to illustrate how the fine structure contributes to the emergence of 1/f fluctuation. A comprehensive analysis encompassing these aspects will be presented in our forthcoming publications.

To refine the Power Spectrum Density (PSD), we introduce the following effects: a) each eigenfrequency labeled by n and l possesses a finite width, and b) the degeneracy in m is resolved by the solar rotation.

a) Resonant modes are typically modeled by the Lorentzian distribution,

$$R[\omega] = \frac{1}{\left(\frac{\kappa}{2}\right)^2 + (\omega - \Omega)^2}, \tag{4}$$

where Ω is the fiducial resonance frequency, and κ characterizes the sharpness of the resonance. This function represents the frequency distribution density associated with the fiducial frequency Ω. The inverse function (tangent) of the cumulative distribution function (hyperbolic tangent) generates this distribution from the Poisson random field.

b) Solar rotation resolves the degeneracy in m by breaking the spherical symmetry of the system. Although the details are intricate, a rough estimate is provided by the resolved frequency [16] in the lowest perturbation in Ω/ω $(\ll 1)$,

$$\omega_{nlm} = \omega_{nl} + \frac{m}{l(l+1)}\Omega, \tag{5}$$

where ω_{nl} is the degenerate eigenfrequency, and $\Omega = 4.3 \times 10^{-7} Hz$ is the frequency associated with solar rotation. The coefficient of Ω is chosen approximately according to [16].

These effects are implemented through a specific process. Initially, we construct wave data by superposing N sinusoidal waves with eigenfrequencies after eliminating the degeneracy in m. Additionally, we superimpose M resonant waves with frequencies proximate to the fiducial frequency, following the distribution in Eq.(4). The fully superposed wave is defined as:

$$\Phi(t) = \sum_{n=1}^{N} \sum_{i=1}^{M} \sin\left(2\pi(1 + c\tan(\xi_i))\Omega_n t\right), \tag{6}$$

where the parameter $c = \kappa/\Omega$ represents the relative line width for each eigenfrequency. The random variable ξ_i, ranging in $[0, \pi/2]$, generates the frequency distribution through $R(\omega)$ in Eq.(4). While c actually depends on each n, for simplicity, we use $c = 0.01$. We utilized data [15], limiting M to 100 and N to 100.

As before, the power spectrum density (PSD) of the bare $\Phi(t)$ exhibits no signal in the low-frequency region. However, taking the absolute value $|\Phi(t)|$ or applying arbitrarily set threshold data produces 1/f fluctuations (details in the caption of Fig.4). These square operations and thresholding essentially function as a demodulation of the original signal. Consequently, the 1/f fluctuation becomes evident only after demodulation and proves to be quite robust. Figure 4 right illustrates the PSD of the thresholded data, demonstrating an approximate 1/f fluctuation with a power index of-1.2, covering a frequency range extended down to 2×10^{-7}Hz. This range partially coincides with the observed range below 10^{-3} Hz. In our future study, further refinement of the PSD analysis is intended, incorporating finer structures of eigenfrequencies, decay times, and deviations of the Sun from spherical symmetry. The introduction of gravitational modes, alongside pressure modes, which operate in much lower frequency domains, is also of interest.

In the preceding discussion, we superimposed the eigenmodes of the Solar Five-minute Oscillation (SFO) to obtain amplitude modulation and pink noise, explaining the observed pink noise in solar flares. However, a more direct examination of the bare data of SFO before decomposition into eigenmodes is warranted. This implies that the resonance of SFO directly yields pink noise. Initially, we utilize SOHO-GOLF data on the fluctuations of the time elapsed $T(t)$ for the waves to circumnavigate the Sun [18]. Details are expounded in [19], with the data spanning about 16.5 years and an interval of 80 seconds, albeit with some data-missing periods.

We commence by calculating the Power Spectrum Density (PSD) of the original data $T(t)$. The result is depicted in the left graph of Fig.5. A prominent peak appears around 3×10^{-3}Hz, corresponding to the typical five-minute mode of solar oscillation. From there, a partial power-law behavior is observed toward 6×10^{-6}Hz with an index of about-1, followed by a flat behavior at lower frequencies. This partial pink noise may stem from instrumental origins [20], potentially not reflecting genuine solar properties.

Conversely, when taking the absolute value of the data, the pink noise region in PSD extends toward the lowest frequency limit, as illustrated in Fig.5 right. The behavior of the SOHO-GOLF data $T(t)$ is precisely the same as the

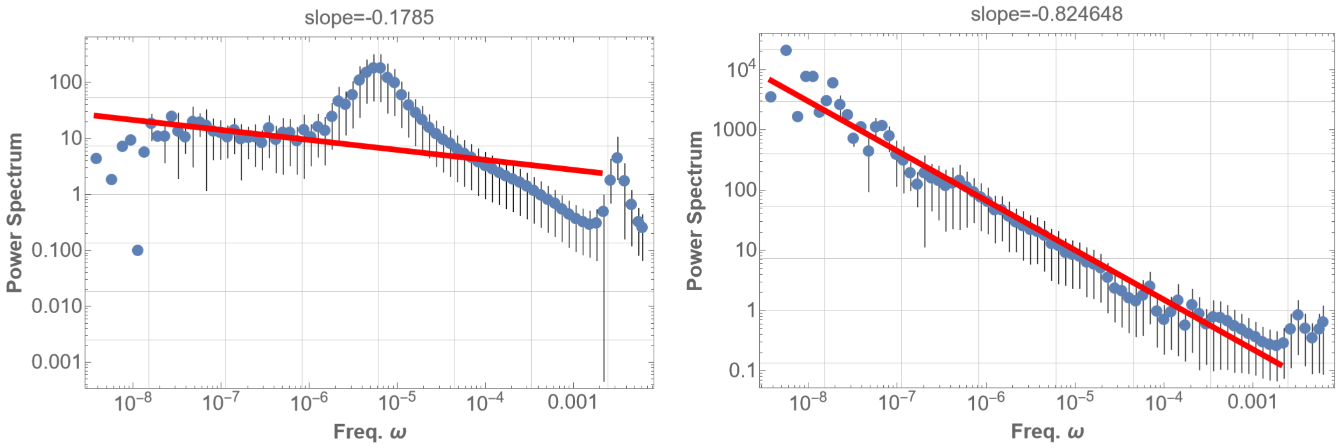

Figure 5. left) The Power Spectrum Density (PSD) is presented for the SOHO-GOLF 16.5-year data, focusing on the time elapsed T(t) for the waves circumnavigating the sun [18]. The rightmost peak corresponds to the typical five-minute mode of the Solar Five-minute Oscillation (SFO) at 3×10^{-3} Hz.
right) The PSD is shown for the absolute value of the original data, revealing pink noise with a slope of -0.8 over approximately six digits.

sound data of orchestra music [21] or the sound of a big bell. While the sound wave amplitude time-sequence data itself does not exhibit pink noise, the square of the amplitude showcases apparent pink noise. These operations of squaring or taking the absolute value naturally correspond to the demodulation process, revealing the encoded pink noise.

This forms part of the rationale behind our belief that solar eigen-oscillations contribute to the pink noise observed in solar flares.

An alternative analysis involves ground-based data from BiSON [22, 23], measuring radial velocity from January 1985 to January 2023 (All sites, Optimized for Quality). The PSD of the original bare data does not exhibit pink noise. However, taking the absolute value of the original data results in clear pink noise with a slope of -0.71, although this index is slightly larger than the right side of Fig.5. This discrepancy is likely influenced by artificial peaks corresponding to the periods of the Earth's spin and rotation at 1.1×10^{-5} Hz and 3×10^{-8}Hz.

A variety of demodulation methods has been considered in association with this BiSON data manipulation. In our previous analysis, we focused on the absolute value of the data, but it is noteworthy that other manipulations can also be employed to extract pink noise. For instance, the squared data exhibits pink noise, while the pink noise disappears in the cubed data. However, when the data is raised to the fourth power, pink noise reappears. These findings strongly indicate that the amplitude-modulated pink noise emerges after specific demodulation processes.

Similar phenomena are often observed in sound systems. For example, we examined sound data collected at the water-harp cave (Suikinkutsu) at HosenIn Temple in Kyoto [24]. The sound is generated by the perpetual impact of water drops on the water surface in the two-meter Mino-yaki pot underground [25]. Although the original sound data barely shows pink noise, the squared data from this sound source clearly displays $1/f$ fluctuation with an index of -0.80 for four digits. The resonator, in this case, is presumed to be the Mino-yaki pot.

V. TIMING STATISTICS

In our analysis of Sec.II, we identified pink noise in the time series of solar flare timing and speculated that this characteristic might be indicative of the low-energy trigger for solar flares. In seismic activity, which also exhibits pink noise[9], the time series of earthquake occurrences is often described by the Weibull distribution function [26, 27]:

$$f(x) = \frac{\alpha}{\beta}\left(\frac{x}{\beta}\right)^{\alpha-1}\exp\left(-\left(\frac{x}{\beta}\right)^{\alpha}\right). \tag{7}$$

We now explore the extent to which the solar flare timing time sequence is characterized by the Weibull distribution and its potential connection to pink noise.

Upon examination of the GOES data [12] used in our analysis, we discover that, in the logarithm of the time interval, it follows the Weibull distribution, as illustrated in Fig. 6 left. The best-fit parameters are $\alpha = 5.3$ and

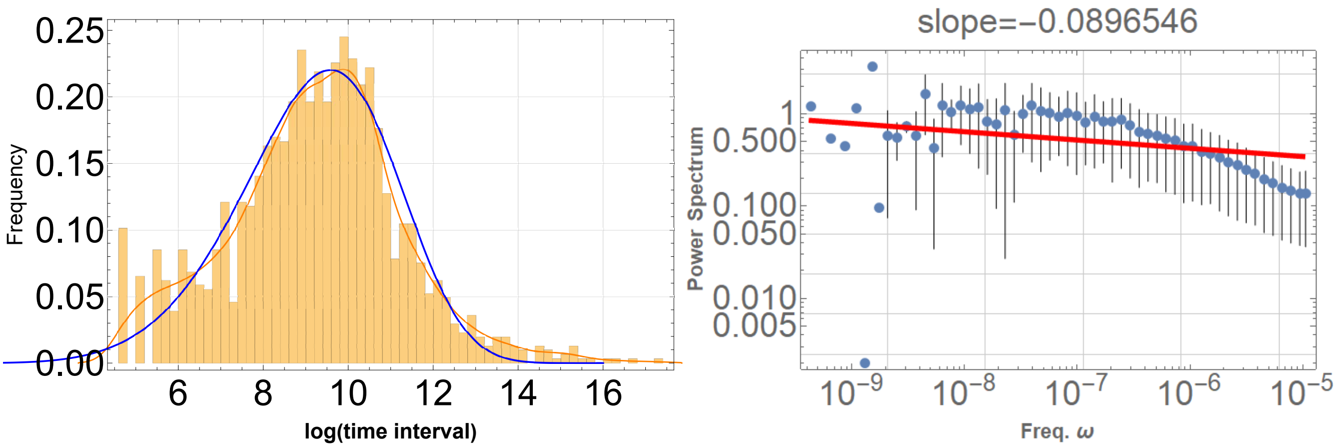

Figure 6. left) Orange bars represent the frequency distribution of the logarithm of the time intervals between solar flare occurrences. The blue line denotes the Weibull distribution with the parameters $\alpha = 5.3$ and $\beta = 9.9$, best fitting the solar flare data.
right) The PSD is presented for artificial data with random time intervals generated by the best-fit Weibull distribution. It is evident that the PSD of this artificial data is flat, indicative of a random distribution.

$\beta = 9.9$. Therefore, similar to seismic activity, the statistical distribution of solar flare timing can be effectively characterized by the Weibull distribution. The question then arises: to what extent does this Weibull distribution characterize the pink noise property?

We have checked that a time sequence simply following the Weibull distribution does not exhibit pink noise; the Power Spectrum Density (PSD) becomes flat in the low-frequency range, as depicted in Fig. 6 right. Consequently, the pink noise observed in solar flare timing is independent of the Weibull distribution, as the behavior observed in seismic cases. This property is readily understood; the time sequence, constructed with randomly chosen intervals according to Weibull distribution statistics, lacks long correlation times. In contrast, pink noise inherently possesses long correlation times, thereby manifesting as a low-frequency property.

VI. ROBUSTNESS AND INHERITED PINK NOISE

We have delved into the origin of pink noise in solar flares, applying the overarching concept that the accumulation of frequencies in numerous waves leads to beat or amplitude modulation. Frequency accumulation is achieved through resonance, and we have successfully explored this concept using the Solar Five-minute Oscillations (SFO), which are consistently resonating. It is plausible that magnetic reconnection serves as the demodulation (DM) of this amplitude modulation (AM), resulting in pink noise in solar flares. If this holds true, the combination of SFO (as AM) and magnetic reconnection (as DM) may generate pink noise beyond solar flare in other extended regions of the solar neighborhood.

For instance, if magnetic reconnection induces solar wind through jetlets [31], triggered by SFO, then the solar wind [32] may also exhibit pink noise. Indeed, pink noise in the solar wind has been observed over many years[33–35]. SFO has been detected in the solar corona [30], suggesting the potential observation of pink noise in that context as well.

Furthermore, the solar wind may interact with the Earth's atmosphere, inducing chemical reactions leading to the production of NO_3^- isotope. This isotope could then become embedded in Antarctic ice cubes. This work is in progress [36]. If solar wind and solar flares influence the Earth's surface, then the sea surface temperature may also exhibit pink noise [37], similar to the case of seismic activity.

In seismic activity, which also displays pink noise, seismic events inherit a pink noise pattern, potentially resulting from amplitude modulation due to resonance with Earth Free Oscillation (EFO) in the lithosphere [9]. This EFO may further contribute to the pink noise observed in the time sequences of volcano eruptions and the fluctuation of the Earth's rotation axes [38].

Including the above robustness of pink noise, we compare solar flares and earthquakes in Table I. This table is preliminary and will be finalized in our forthcoming study.

	Solar Flare	Eathquakes
AM (resonator)	Solar Five-minute Oscillation (SFO)	Earth Free Oscillation (EFO)
DM	magnetic reconnection	fault rupture
PSD total data	flat (GOES16, RHESSI)	flat (USGS)
PSD low-energy	pink (GOES16, RHESSI)	pink (USGS)
PSD timing	pink (GOES16, RHESSI)	pink (USGS)
PSD superposed eigenmodes	pink (JSOC)	pink (T. G. Masters, R. Widmer)
Weibull Distribution	yes: $\alpha = 5.3$ and $\beta = 9.9$ (GOES16)	yes: $\alpha = 6.3$ and $\beta = 7.63$ (USGS)
PSD resonator	pink (SOHO-GOLF, BiSON)	?
inherent phenomena	solar wind, sunspot number, nitrate, SST,cosmic ray[37]	volcano eruption, rotation axes[38]

Table I. Similarity of solar flares and earthquakes from the view point of pink noise.
This is a tentative table, and the detail will be reported soon by the authors.

VII. CONCLUSIONS AND PROSPECTS

In conclusion, our investigation has identified pink noise in the solar flare time series, and we have partially elucidated its origin by applying our proposed mechanism: pink noise emerges from the resonance of the Solar Five-minute Oscillations via amplitude modulation and demodulation.

Our GOES data analysis of the solar flare time sequence has revealed distinct low-frequency properties. Specifically, the power spectrum density of low-energy flares ($E \leq E_{mean}$) exhibited 1/f fluctuations, while high-energy flares ($E > E_{mean}$) displayed a flat spectrum. Notably, the time sequence of flare occurrences demonstrated clearer 1/f fluctuations, indicating that low-energy characteristics play a pivotal role in triggering the observed 1/f fluctuations in solar flares. Building on our recent proposal that 1/f noise arises from amplitude modulation and demodulation, we postulated that this modulation is encoded through resonance with the Solar Five-minute Oscillation (SFO) and demodulated via magnetic reconnection.

To test this hypothesis, we constructed a dataset by superposing sinusoidal waves with 2247 eigenfrequencies of SFO. The absolute value of this time sequence marginally exhibited 1/f fluctuations with a power index of -0.5 down to 2×10^{-5}Hz. Further refinement of the data, considering resonance effects and finer structures labeled by m induced by solar rotation, involved adding 100 extra modes generated by resonant Lorentzian distributions for the first 100 eigenfrequencies of SFO after resolving the degeneracy in m. The absolute value of this refined time sequence clearly displayed 1/f fluctuations with a power index of -1.2 down to 10^{-7}Hz, largely overlapping with the observed range of solar flare 1/f fluctuations (power index -1.0 from about 2×10^{-3} Hz down to2×10^{-8}Hz). Thus, our analysis provided partial verification that SFO triggers seismic 1/f fluctuations.

Further investigation into SOHO-GOLF data and BiSON data for velocity fluctuations in the solar atmosphere revealed that the original time sequence of these data barely exhibited 1/f fluctuations, while the absolute values of the time sequence did display clear 1/f fluctuations. This lends additional support to our proposition: SFO is the origin of solar flare 1/f fluctuations.

Additionally, our examination of the time sequence of solar flare occurrences revealed adherence to a Weibull distribution. However, an artificial time sequence composed from the Weibull distribution barely exhibited 1/f fluctuations, suggesting that the Weibull distribution does not fully characterize solar 1/f fluctuations.

Lastly, a comparison between 1/f fluctuations in solar flares and earthquakes demonstrated remarkable similarities [28][29]. Furthermore, we propose that a comparable analysis may be applicable to the activity of a black hole/disk system, replacing the resonator SFO with the Quasi-Periodic Oscillation of a black hole. This broader perspective underscores the potential universality of the proposed amplitude modulation and demodulation mechanism in diverse astrophysical phenomena.

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